

# ***Strategies for Improving Tier I and II Mathematics Instruction in a Response to Instruction and Intervention Model***

Response to Instruction and Intervention in PA:  
An All Education Standards Aligned Initiative

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# Overview

- Instructional Foundations
  - NMAP 2008 Final Report
  - IES Rtl Math Practice Guide
- Tier I and II Instructional Supports
  - Scaffolding Problem Solving
  - Facilitating Thinking a louds
  - Spaced Learning Overtime
  - Interleave Worked out Solutions
- Conclusion



# Foundations for Success

## National Mathematics Advisory Panel

Final Report, March 2008



Select Slides taken from the NMAP-Final Report  
Presentation available at: <http://www.ed.gov/MathPanel>

# Learning Processes

- To prepare students for Algebra, the curriculum must **simultaneously** develop **conceptual understanding**, **computational fluency**, **factual knowledge** and **problem solving skills**.
- Limitations in the ability to keep many things in mind (**working-memory**) can hinder mathematics performance.
  - **Practice** can offset this through automatic recall, which results in less information to keep in mind and frees attention for new aspects of material at hand.
  - Learning is most effective when **practice is combined with instruction** on related concepts.
  - Conceptual understanding **promotes transfer** of learning to new problems and better long-term retention.



# Instructional Practices

Research on students who are low achievers, have difficulties in mathematics, or have learning disabilities related to mathematics tells us that the effective practice includes:

- Explicit methods of instruction available on a regular basis
- Clear problem solving models
- Carefully orchestrated examples/ sequences of examples.
- Concrete objects to understand abstract representations and notation.
- Participatory thinking aloud by students and teachers.



# IES Rtl Math Practice Guide

## Focus on 1 of 8 Recommendations

- #3: Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.



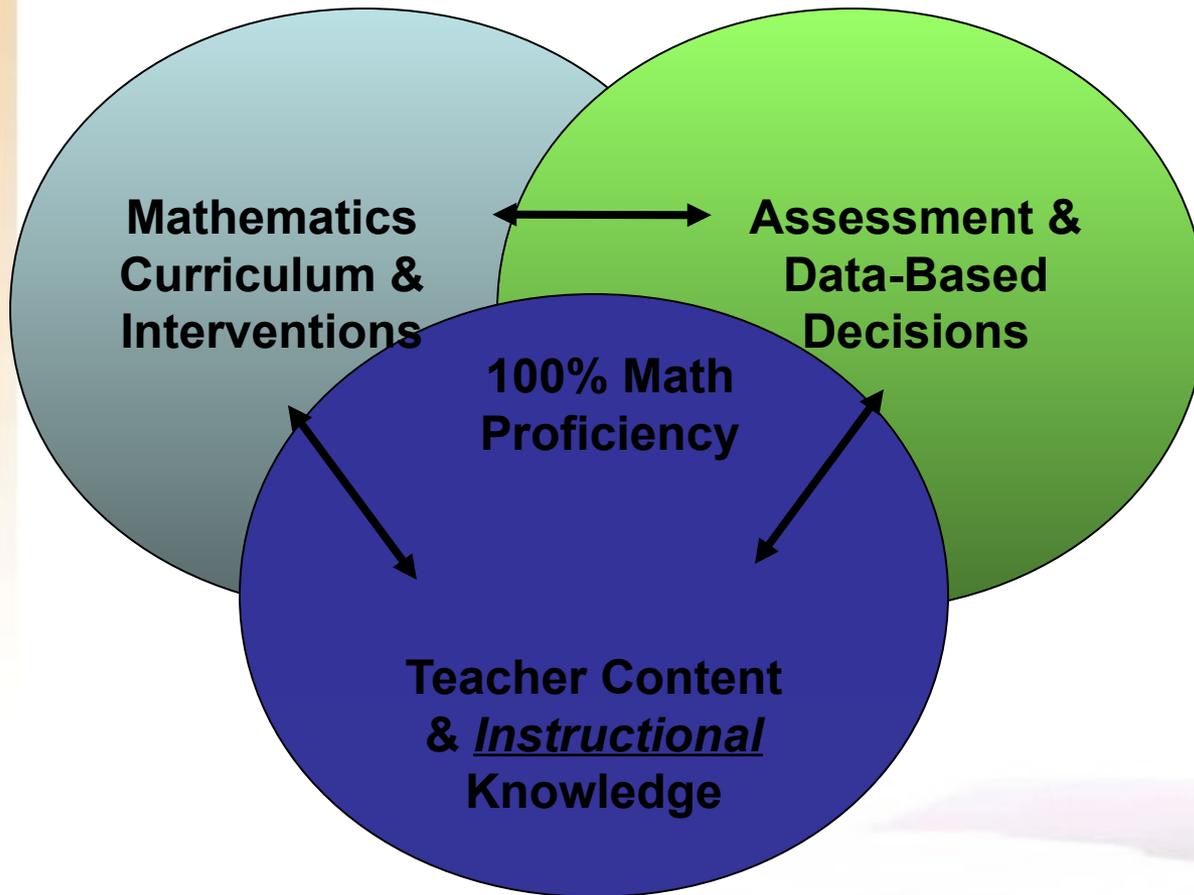
# IES Rtl Math Practice Guide

## Focus on 1 of 8 Recommendations

- #3: Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.



# Components of Effective Mathematics Programs



http://www.pattan.net/files/rti/parentguide.pdf - Windows Internet Explorer

http://www.pattan.net/files/rti/parentguide.pdf

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Connect Select

http://www.pattan.net/files/rti/parentguide.pdf

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A Parent's Guide to  
Response to Intervention (RtI) in Pennsylvania

Tier 3: Intensive Intervention  
For students significantly below grade level

Tier 2: Targeted Group Intervention  
For students at academic or behavioral risk  
8-monthly Progress Monitoring

Tier 1: Core Instruction  
For all students  
Universal Screening and Benchmark Assessments  
(2 to 3 times per year)

Tier 3  
Interventions  
for a  
Few Students

Tier 2  
Interventions  
for  
Some Students

Tier 1  
Foundation  
Standards Aligned Instruction

Increasing Time & Intensity of Interventions

ALL INTERVENTIONS ARE RESEARCH-BASED AND STANDARDS ALIGNED

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RTI Institute E-mail Request.pdf - Adobe Acrobat Pro

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yourself, they will have also heard 3 other experts speak to math, literacy, assessment for learning, and behavior. We envision that you and the other three experts will pose challenging questions to all teams, then "drift" among them to provide assistance through participation as "a panel". Finally, we will close the morning session by bringing all back together to close out any other lingering questions.

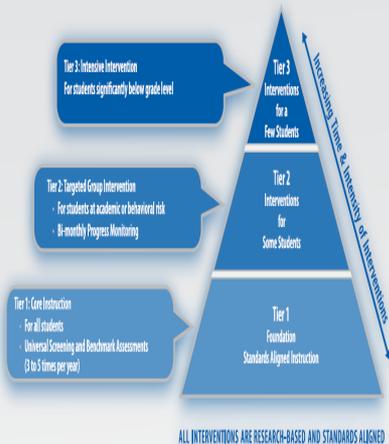
6/15, day 2		
1:15 - 2:45PM	Gain a Deeper Understanding	<ul style="list-style-type: none"> <li>Review all effective instructional variables related to math</li> <li>Could choose 2-3 to go a little deeper into (use examples or nonexamples), i.e.               <ul style="list-style-type: none"> <li>What does the use of a visual representation look like in math?</li> <li>Use of Student Think Alouds in math</li> <li>Teach students using explicit instruction on a regular basis.</li> </ul> </li> </ul>
3-4:30PM	Gain a Deeper Understanding	Repeat of the above
6/16, day 3		
8:30 - 9:00AM	Experts offer challenges to Teams	
9:10-10:50AM	Team Time	Bringing learning together via teams - expert panel (visit each strand/group and pop in for a Q/A and move to next team and pop in for a Q/A
11:00-11:30AM	Time with Experts / Q	

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# MORE & BETTER (Effective) INSTRUCTION

## A Parent's Guide to Response to Intervention (RtI) in Pennsylvania



6/15, day 2

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# **General Components: Form the Basis of Effective Math Instruction**

- 1. Engaged Time**
- 2. Student Success Rate**
- 3. Content Coverage & Opportunity to Learn**
- 4. Grouping for Instruction**
- 5. Scaffolded Instruction**
- 6. Addressing Forms of Knowledge**
- 7. Activating & Organizing Knowledge**
- 8. Teaching Strategically**
- 9. Making Instruction Explicit**
- 10. Making Connections**



# General Components: Form the Basis of Effective Math Instruction

1. Engaged Time
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10. Making Connections



# 5. Scaffolded Instruction

*Instructional scaffolding* is a process in which a teacher adds supports for students to enhance learning and aid in the mastery of tasks.



# 5. Scaffolded Instruction

- temporary and adjustable support
- reduce task to fewest steps
- initial explicit demonstration
- promote student elaboration
- promoting cueing and fading of cues
- scaffolding and explicit instruction



# Instructional Scaffolding

- **3 Levels of Instructional Scaffolding**
  - Content
  - Task
  - Material



# 3 Levels Instructional Scaffolding

- **Content Scaffolding**
  - the teacher selects content that is ***not distracting*** (i.e., too difficult or unfamiliar) for students when learning a new skill.
  - allows students to ***focus on the skill being taught***, without getting stuck or bogged down in the content
- **3 Techniques for Content Scaffolding**
  - Use Familiar or Highly Interesting Content
  - Use Easy Content
  - Start With the Easy Steps



# Example of Content Scaffolding

- **Math Word Problems Strategy Instruction**

- Remove irrelevant information
- Include answer in the problem (i.e., no question)
- Allows students to focus in process of strategy

- **For example:**

- Robert planted an oak seedling. It grew 10 inches the first year. Every year after it grew  $1\frac{1}{4}$  inches. How tall was the oak tree after 9 years?
- An oak seedling grew 10 inches in the first year. Every year after it grew 1 inch. After 9 years the oak tree was 18 inches tall.



# Breakout Activity (Handout #9)

- Write a number sentence for the word problem:
  - An oak seedling grew 10 inches in the first year. Every year after it grew 1 inch. After 9 years the oak tree was 18 inches tall.



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$$10 + 1+1+1+1+1+1+1+1=18$$

OR

$$10 + (1)(8) = 18$$

OR

$$10 + (1)(9-1) = 18$$



# Breakout Activity (Handout #9)

- Write a number sentence for the word problem
  - An oak seedling grew 25 feet in the first year. Every year after it grew 5 feet. After 4 years the oak tree was 40 feet tall.



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  - An oak seedling grew 25 feet in the first year. Every year after it grew 5 feet. After 4 years the oak tree was 40 feet tall.

$$25 + 5 + 5 + 5 = 40 \text{ feet tall}$$

OR

$$25 + 5(3) = 40 \text{ feet tall}$$

OR

$$25 + (5)(4-1) = 40$$

- Rewrite number sentence with all variables.



# Breakout Activity (Handout #9)

- Write a number sentence for the word problem
- Rewrite number sentence with all variables.
- Now solve this problem
  - An oak seedling grew 4 meters in the first year. Every year after it grew 2 meters. After 7 years, how tall was the oak tree?



# Breakout Activity (Handout #9)

- Solve the more complex problem
  - Robert planted an oak seedling. It grew 10 inches the first year. Every year after it grew  $1\frac{1}{4}$  inches. How tall was the oak tree after 9 years?
- Scaffolded Instructional Progression
  - This is how teachers can help students progress from simple tasks to more complex problem solving tasks.



# Instructional Scaffolding

## • Task Scaffolding

- **Specify the steps** in a task or instructional strategy
- **Teacher models the steps** in the task, verbalizing his or her thought processes for the students.
- the **teacher thinks aloud and talks** through each of the steps he or she is completing
- Even though students have watched a teacher demonstrate a task, it does not mean that they actually understand how to perform it independently



# Approaching Word Problems

- **Explicit modeling of cognitive and metacognitive strategies**
- Steps
  - Read for understanding
  - Paraphrase in your own words
  - Visualize a picture or diagram
  - Hypothesize a plan to solve the problem
  - Estimate or predict the answer
  - Compute or do the arithmetic
  - Check to make sure everything is correct



# Providing Structure (Handout #10)

Who or what is involved in the action	
Math vocabulary used	
Paraphrase the question / problem	
Write equation to obtain solution	
<i>Explain equation reasoning</i>	
<i>Explain solution</i>	

# Instructional Scaffolding

## • Material Scaffolding

- Material scaffolding involves the use of written prompts and cues to help the students perform a task or use a strategy.
- This may take the form of **cue sheets** or **guided examples** that list the steps necessary to perform a task.
- **Students can use these as a reference, to reduce confusion and frustration.**
- The **prompts and cues should be phased** out over time as students master the steps of the task or strategy.



# Example of Material Scaffolding

- Concepts Maps—better to use a few rather than 50 different concepts maps
- Posters and bulletin boards are other examples. Remember they must be **faded over time**



# Concept Maps and Graphic Organizers Reinforce Connections

Algebra Case Study Unit Part 2.pdf - Adobe Reader

File Edit View Document Tools Window Help

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- Graphic organizers can be used to assist students in breaking down the steps of problems.

Multiply the base by itself 6 times

Exponent

Base

$4^6$

$4 \times 4 \times 4 \times 4 \times 4 \times 4$

$16 \times 16 \times 16 =$

4096

RESOURCES...

Ives, B., & Hoy, C. (2003). Graphic organizers applied to higher-level secondary mathematics. *Journal of Research in Science Teaching*, 40(2), 26-51.

# Example of Material Scaffolding

- *Guided examples:* A step-by-step instructional guide for how to apply a strategy or complete a task.

**How to Add Fractions with Different Denominators**  
(NEVER ADD THE DENOMINATORS!)

$$\frac{1}{4}$$

+

$$\frac{1}{3}$$

---

**STEP 1:** Find the Least Common Multiple of the two denominators.

**STEP 2:** Rename the fractions so that each has the LCM as the denominator. This is called the common denominator. For each fraction, find out what the original denominator was multiplied by to get the common denominator and multiply the numerator by the same number.

**STEP 3:** You should now have two fractions with the same denominator. Add the numerators and use the common denominator.

Name \_\_\_\_\_

**Reducing Fractions**

To reduce a fraction like  $\frac{6}{10}$ , follow these steps.

**STEP 1:** List all the factors of the numerator and the denominator

6: 1, 6, 2, 3  
10: 1, 10, 2, 5

**STEP 2:** Put a box around the numbers that are on both lists.  
The largest number is the Greatest Common Factor

6:  $\boxed{1}$ , 6,  $\boxed{2}$ , 3  
10:  $\boxed{1}$ , 10,  $\boxed{2}$ , 5

GCF: 2

**STEP 3:** Divide the numerator and the denominator by the GCF  
The fraction you are left with is the reduced fraction.

$$\frac{6 \div \boxed{2}}{10 \div \boxed{2}} = \frac{3}{5}$$


# Breakout Activity (Handout #11)

- Scaffolding a Task
  - Describe what happens to the circumference and area when the radius doubles?



# Scaffolding

- How much scaffolding is necessary?
- **BOTTOM LINE:**

**As much as the  
students require to  
learn and be  
successful!**



## 9. Making Instruction Explicit

- a. Make goals, objectives, and expectations explicit
- b. Make instructional content explicit
- c. Make the structure of the lesson explicit



# Instructional Practices

Research on students who are low achievers, have difficulties in mathematics, or have learning disabilities related to mathematics tells us that the effective practice includes:

- Explicit methods of instruction available on a regular basis
- Clear problem solving models
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- Concrete objects to understand abstract representations and notation.
- Participatory thinking aloud by students and teachers.



# Explicit Instruction

- **Six Critical Features of Explicit Instruction**

1. Daily Reviews
2. Presentation of New Content
3. Guided Practice
4. Explicit feedback and Correctives
5. Independent Practice
6. Weekly and Monthly Reviews

See Handout #13 and 13a



# Weekly and Monthly Reviews

- Much of teaching is about helping students master new knowledge and skills and then helping students **NOT** to forget what they have learned.
- Facilitate learning and remembering information
- Work Smarter NOT Harder!



# Recommendations

1. Space learning over time
2. Interleave worked example solutions and problem-solving exercises



# Recommendation #1: Space learning over time

- Arrange for students to have **S**paced *I*nstructional **R**eview (*SIR*) of key course concepts (Big Ideas)
  - At least 2 times
  - Separated by several weeks to several months
- Why:
  - Helps student remember key facts, concepts, and knowledge



# Spaced Learning Overtime (Handout #14)

Components of Explicit Instruction Checklist - Microsoft Word

Yes	Features of Explicit Instruction	How/When/Where/Who?
<input type="checkbox"/>	Daily Reviews	
<input type="checkbox"/>	Presentation of New Content	
<input type="checkbox"/>	Guided Practice	
<input type="checkbox"/>	Explicit Feedback and Correctives	
<input type="checkbox"/>	Independent Practice	
<input type="checkbox"/>	Weekly and Monthly Reviews	
<input type="checkbox"/>	Recommendations from the NYMAP Final Report	
<input type="checkbox"/>	Clear problem solving models	
<input type="checkbox"/>	Clearly orchestrated examples/sequences of examples	
<input type="checkbox"/>	Concrete objects to understand abstract representations and notation	
<input type="checkbox"/>	Participatory thinking aloud by students and teachers	

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<http://www.jshorpe.com/edu/2009/04/27/14/>

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Spaced Instructional Review Planning Sheet

Block (Date)	Big Ideas Covered	Problematic Areas	Problematic Areas Targeted for SIK	Done and Instructional Time Allotted (30-40 minutes)
1. 2. 3. 4. 5. 6.		1. 2. 3. 4.	1. 2.	
1. 2. 3. 4. 5. 6.		1. 2. 3. 4.	1. 2.	
1. 2. 3. 4. 5. 6.		1. 2. 3. 4.	1. 2.	
1. 2. 3. 4. 5. 6.		1. 2. 3. 4.	1. 2.	

Notes: This spreadsheet has been Teacher assessment (final 4 columns), Progress Monitoring (last column), Data Assessment, and other sources of information (teacher's responses). Table intended for open-ended responses.

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Why is it that multiplying two negative integers gives a positive result but multiplying two positive integers cannot give a negative result?

Answer

user-generated content: daplamar

Q: Why is it that multiplying two negative integers gives a positive result but multiplying two positive integers cannot give a negative result?

In: Math, Algebra, Arithmetic

A:

It is well known that if you multiply a positive number (it doesn't have to be an integer) by another positive number the result is always a positive number.  
 For example  $(+2,3) \times (+2) = +4,6$

Similarly, at school you learn that if you multiply a positive number by a negative number the result is always a negative number.  
 For example  $(+19) \times (-2) = -38$

When you multiply two negative numbers you must get a positive result because of the way negative numbers "work", but why that is so is not usually explained at junior or high school. You learn why as part of learning about complex numbers in higher math at technical college or university.

To be very brief, it is all to do with the behaviour of the operator  $-$ . (For more information about it please see the answer to the related question shown below.)

Because  $i^2$  is defined to be equal to  $-1$  then it follows that  $i^2 \times i^2 = +1$  for the following reason: multiplying something by  $i^2$  means "apply the operator i twice" so, if you start with a positive number, multiplying it by  $i^2$  makes that positive number into a negative number. If you multiply that negative number by  $i^2$  again you make the result into a positive number.

Overall you are applying the operator i four times so you can say that  $i^2 \times i^2$  (which is the same as  $i^4$ ) means "apply the operator i four times" and doing that gives a result which is a positive number.

For example  $(-1) \times (-2) = +2$  could be written as:  
 $[(+1) \times i^2] \times [(+2) \times i^2] = +2 \times [(i^2 \times i^2)] = +2$   
 or even as  $(+1) \times i^2 \times (+2) \times i^2 = +2 \times i^4 = +2$

Online Master's Degrees  
 Compare Online Schools and Degrees. Financial Aid Available.  
[www.EducationDegreeSource.com](http://www.EducationDegreeSource.com)

Math Learning Center  
 The most effective math learning center uses the Kumon Method  
[www.kumon.com/math-learning-center](http://www.kumon.com/math-learning-center)

The Math Solution LLC  
 Need online math help? View free video math lessons. Visit us today!

# Recommendation #1 (con't)

- Caution: some important content is automatically reviewed as the learner progresses through the standard curriculum
  - For example: Students use single digit addition nearly every day in second grade
- This recommendation applies to important knowledge and skills that are not automatically reviewed



# Recommendation #1 (con't)

- Make sure important and essential curriculum content is reviewed at least 3-4 weeks after it was initially taught.
- Benefits of a delayed review is much greater than the same amount of time spent reviewing shortly after initial instruction (Rohrer & Taylor, 2006).



# Recommendation #1 (con't)

1. Use class time to review important curriculum content
  - For example, every other week a 4th grade teacher spends half the class reviewing an important math skill taught in the previous 3-4 weeks (i.e., estimation, LCD, fractions)
2. Use homework assignments as opportunities for students to have spaced practice of key skills and content
  - For example, in every homework assignment a math teacher intentionally includes a few problems covering material presented in class 1 or 2 months ago
3. Give cumulative midterm and final exams
  - Provides student incentives to study all course material at widely separated points in time.



## Recommendation #2: Interleave Worked Example

- Interleave worked example solutions and problem-solving exercise
- Literally, alternate between worked examples demonstrating one possible solution path and problems that the student is asked to solve independently
- This can markedly enhances student learning



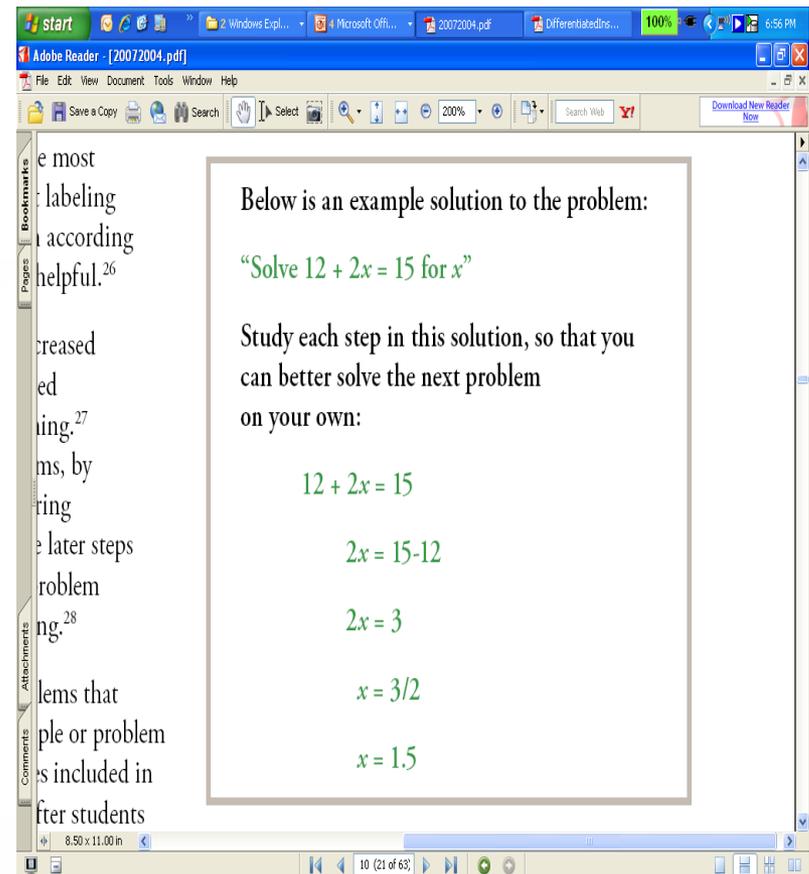
# Recommendation #2: Interleave Worked Example

- Typical Math Homework assignment
  - Pg. 155 #1-21 odd
- Students are required to solve all problems.



# Recommendation #2: Interleave Worked Example

- Interleaved Homework assignment
  - Pg 155 1-10 (all)
  - Odd problems



The screenshot shows a PDF document in Adobe Reader. A text box highlights a worked example solution for the equation  $12 + 2x = 15$ . The text in the box reads: "Below is an example solution to the problem: 'Solve  $12 + 2x = 15$  for  $x$ ' Study each step in this solution, so that you can better solve the next problem on your own:  $12 + 2x = 15$ ,  $2x = 15 - 12$ ,  $2x = 3$ ,  $x = 3/2$ ,  $x = 1.5$ ".

# Recommendation #2: Interleave Worked Example

## Other considerations:

1. The amount of guidance an annotation accompanying the worked out examples varies depending on the situation
2. Gradually fade examples into problems by giving early steps in a problem and requiring students to solve more of the later steps
3. Use examples and problems that involve greater variability from one example or problem to the next
  - Changing both values included in the problem and the problem formats.



# Recommendation #2: Interleave Worked Example

- During Whole Class instruction

1. Start off discussion around an already solved problem
  - Pointing out critical features of the problem solution
2. After discussion have students pair off in small groups or work individually to solve a problem (JUST ONE!) on their own
3. Then back to studying an example, maybe one student presents their solution and have others attempt to explain
4. Then after studying the solved example, students are given another problem to try on their own.



# Organizing Instruction and Study Time

- **Remember it's always easier to work smarter NOT harder**



# Summary

- Foundation of Tier I and II Instructional supports
  - Explicit and systematic
  - Scaffolding Supports
    - Content
    - Task
    - Material
  - Space Learning Overtime
  - Interleave Worked Out Solutions



# Improving Our Educational Practices

“If you always do what you have always done, you’ll always get what you’ve always gotten.”

Helen Bernstein



# Questions?

